Multi-physics analysis of electric vehicles (EV) powertrain

System interactions in EV traction motor design

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Motor Design Ltd (MDL)

Motor-CAD Software
- Develop Motor-CAD software for electric motor design
- High level of customer support and engineering know-how
- Motor design software is developed by motor engineers

Consultancy
- Design, analysis & training

Research
- Involved in collaborative government/EU funded research projects:
  - **Concept_e** – Prototype Electric vehicle development with Jaguar Land Rover (JLR)
  - **HVEMS** – High Volume E-Machines Manufacturing Supply Make-Like-Production prototyping facility in the UK with JLR
  - **Tevva** – Design of SRM motors for Trucks
  - **ReFreeDrive** – Rare Earth Traction motors with improved performance and lower cost (Induction and Reluctance Motors)
  - **ELETAD** – Helicopter electric tail rotor
Motor Design Limited & The University of Manchester

- Collaborate with universities worldwide to develop electric machine modelling techniques and create validation data
  - The University of Manchester (UK)
  - The University of Warwick (UK)
  - The University of Nottingham (UK)
  - University of Bristol (UK)
  - University of Cassino (IT)

- Collaboration with The University of Manchester (UoM)
  Collaborative research to model mechanical stress in electrical machines due to high speed effects
Development of an electric vehicle (EV) powertrain is a complex systems problem

Achieving an optimal system design requires evaluation of many different concepts and topologies as well as detailed understanding of the system interactions.

These interactions are typically cross specialism or discipline, involve different teams and often require multi-physics analysis.

Design and simulation tools are crucial to evaluating different design topologies as well as identifying and understanding important system interactions.
Improved System Design Workflow

Motor Design Efficient Toolkit Motor-CAD

Advanced Magnetics Modeling 2D + 3D Analysis

Mechanical Analysis CFD, NVH, Stress

System Validation Control logic

Design → Analysis → Operation

Motor Design

Efficient Toolkit Motor-CAD

Thermal Emag

Advanced Magnetics Modeling 2D + 3D Analysis

Mechanical Analysis CFD, NVH, Stress

System Validation Control logic

3D Physical Validation

Concept Design System Validation
Improved System Design Workflow

Multi-Physics Analysis of E-Machines Over the Full Torque/Speed Operating Envelope

• Integrated software for motor performance analysis

Motor-CAD:

- **EMag**: 2D FEA electromagnetic analysis and loss calculation
- **Therm**: Network/FEA Thermal Analysis
- **Lab**: Fast prediction of efficiency maps and drive cycles

Design → Analysis → Operation
Improved System Design Workflow

Motor-CAD EMag
- Motor Types:
  - BPM (inner & outer rotor)
  - Induction
  - Synchronous reluctance
  - Switched reluctance
  - Synchronous wound field
- Very fast and easy to set-up a design and do complex analysis
- Comprehensively validated

Winding eddy current loss
Improved System Design Workflow

Motor-CAD EMag
- Extensive range of parametrised templates geometries with additional flexible DXF or script based geometry definition
- Fast 2D FEA transient electromagnetic solver combined with analytical models
- Analysis of losses including AC winding losses & magnet eddy currents
- Standard or custom winding designs

Motor Design
Efficient Toolkit
Motor-CAD

Motor - CAD EMag
- Rotor Geometry Created using DXF
- Rotor Geometry Created using Script

Design → Analysis → Operation
Improved System Design Workflow

Motor-CAD Therm: Cooling Types Investigated

Motor-CAD includes models for an extensive range of cooling types:
• TENV: Totally enclosed non-ventilated (Natural convection from housing)
• TEFC: Totally enclosed fan cooled (Forced convection from housing)
• Through ventilation
• TE with internal circulating air (Internal air circulating path, water jacket as heat exchanger)
• Open end-shield cooling
• Water jackets (Axial or circumferential)
• Submersible cooling
• Wet rotor & wet stator cooling
• Spray cooling (e.g. Oil spray cooling of end windings)
• Direct conductor cooling (e.g. Slot ducts with oil)
Improved System Design Workflow

Motor-CAD Therm: Thermal Network
- Thermal and flow network analysis
- 3D network automatically generated
- 20 years of embedded experience in thermal modelling of eMachines

Motor Design Efficient Toolkit
Motor-CAD
Thermal
Emag

Design Analysis Operation
Improved System Design Workflow

Motor-CAD Therm: Steady-State & Transient Thermal Analysis

- Calculation of steady-state or transient thermal performance
- Temperature rises over a complex duty cycle can be solved rapidly and analysed iteratively during the design process
Improved System Design Workflow

Motor-CAD Lab: Virtual Testing Laboratory
Virtual testing including fast calculation of Efficiency Maps/Losses and Duty Cycle Analysis
• Very fast and accurate prediction of the motor electromagnetic and thermal performance over the full torque/speed envelope by use of intelligent loss algorithms
• Automated calculation of optimum phase advance angle for maximum torque/amp or maximum efficiency control
• Suited to applications such as traction applications that have complex duty cycle loads

Efficiency map with drive cycle overlaid

Loss vs Time calculated from efficiency map to be input into thermal model
**Improved System Design Workflow**

**Motor-CAD Lab: Electromagnetic and Thermal Limited Envelope**

**Peak torque envelope**
- Maximum torque/amp or maximum efficiency control

**Continuous torque envelope**
- Co-simulations between electromagnetic model (via flux linkage and loss maps) with thermal model
- Maximum torque at different speeds for a limited winding and rotor temperature
- Thermal transient for a set amount of time that gives a certain maximum winding temperature

This is output matches how electric motors are typically specified. It is very useful to compare these curves for different design variations.
Motor-CAD Lab: Dynamic Operations

- The operation of these machines is very dynamic and considerations of performance across the full torque/speed operating envelope are required.
- Modelling tools need to support this, Motor-CAD is a unique solution on the market for this type of analysis.
- It allows machine efficiency to be optimised over standard operating cycles and sized for a worst-case cycle, giving minimum system size and cost.

Efficient Toolkit

Motor-CAD

Vehicle Speed Profile  
Motor/Generator: Time vs Torque vs Speed  
Motor/Generator: Loss vs Time  
Motor/Generator: Temperature Vs Time

Design  Analysis  Operation
Improved System Design Workflow

Export Model for 3D Simulations

- Partnership with Ansys provides coupling to high power numerical simulation
  - 3D FEA for analysis of end-effects with Ansys Maxwell

3D leakage effects can be important and could worsen motor performance
Export Model for 3D Simulations

Interaction between other powertrain components is handled through coupling with other specialist tools

• Combined NVH response of motor and gearbox using Motor-CAD and Romax designer

• CFD for analysis of heat transfer due to fluid flow with Fluent

• Mechanical stress and vibration analysis with Ansys Mechanical

• Development of stress analysis tool with UoM
Improved System Design Workflow

System Simulations

Vehicle thermal system behaviour
• Co-simulation with GT-Suite

Combined inverter and motor behaviour
• Model export to Simulink Simscape mapping of motor attributes that take in account non-linear motor behaviour

Motor Flux Mapping

Motor Torque Mapping

Motor Model Implementation in Simulink Environment

System Validation

Control logic

Concept Design

System Validation

3D Physical Validation
**Improved System Design Workflow**

**Coupled e-machine and inverter modelling**

Motor behaviour considering

- Time harmonics in current waveform
- $L_d$, $L_q$, $\lambda_m$ model with saturation and positional variation

1. **Input current waveform** – initially ideal
2. **Calculate $L_d(\theta)$, $L_q(\theta)$, $\lambda_m(\theta)$**
3. **Simulate inverter circuit including DC link and control loops using calculated motor inductances**
4. **Feed updated current waveforms back into FEA simulation**

**Convergence loop**

Design ➔ Analysis ➔ Operation
Tutorial Overview

Motor Design
Trade-off Analysis for a BEV traction application

1. eMachine Comparison - PM, IM, Sync
2. Windings Comparison - Hairpin vs Stranded
3. Cooling Comparison - Water Jacket, Internal Air and Oil Spray

Mechanical Analysis

4. NVH Analysis - Behaviour of Motor + Gearbox
5. Mechanical Stress Analyses (Problem formulation and Solution)
**Tutorial Overview**

**Motor Design**

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**Mechanical Analysis**

NVH and Mechanical Stress

4. NVH Analysis - Behaviour of Motor + Gearbox
5. Mechanical Stress Analyses (Problem formulation and Solution)
Many motor types and topologies have been developed recently, as seen by the wide range of EV traction motor designs on the market.
1. eMachine Comparison

- Using published teardown data for Nissan LEAF motor
- Developed models to validate & demonstrate software tools for modelling traction applications
1. eMachine Comparison

Comparison of PM, IM and Sync traction machine types

- Same outer diameter
- Same peak performance requirements
- Different axial lengths
- Same water jacket cooling

Specifications

<table>
<thead>
<tr>
<th>Feature</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Torque</td>
<td>350 Nm</td>
</tr>
<tr>
<td>Peak Power</td>
<td>150 kW</td>
</tr>
<tr>
<td>DC Link Voltage</td>
<td>400 Vdc</td>
</tr>
<tr>
<td>Max Current</td>
<td>500 Arms</td>
</tr>
<tr>
<td>Stator Outer Diameter</td>
<td>250 mm</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>12,000 rpm</td>
</tr>
</tbody>
</table>

Cooling System

<table>
<thead>
<tr>
<th>Feature</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Temperature</td>
<td>65 degC</td>
</tr>
<tr>
<td>Coolant Flow Rate</td>
<td>6.5 l/min</td>
</tr>
<tr>
<td>Coolant</td>
<td>EGW 50/50</td>
</tr>
</tbody>
</table>

Cooling channels over active machine section only
1. eMachine Comparison

**Design**

**Brushless PM machine**

- 48 slot 8 pole IPM
- Double layer magnet – similar to BMW i3
- N42UH magnet
- M250-35A steel
- 250mm OD
- Multi-stranded stator windings
- Step skewed rotor on the market
1. eMachine Comparison

**Brushless PM machine**

- Single layer winding
- Coil pitch - 5 slots
- 6 turns per coil, with 15 strands per turn
- 40% copper slot fill
- 2 parallel paths per phase
1. eMachine Comparison

Brushless PM machine

- Back emf waveform at 500rpm
- 3 slices combine to give sinusoidal waveform
- Torque waveform at 350Nm
- 3 slices combine to minimise torque ripple
- Torque ripple = 4.4%
1. eMachine Comparison

Induction machine (IM)

- 72 slot 84 bar
- 6 pole
- Copper rotor
- M250-35A
- 250mm OD
- Multi-stranded winding
- 5° mech rotor bar skew
Design

1. eMachine Comparison

Induction machine (IM)

- Single layer winding
- Coil pitch - 11 slots
- 3 turns per coil, with 15 strands per turn
- 40% copper slot fill
- 2 parallel paths per phase
1. eMachine Comparison

**Induction machine (IM)**

- Torque waveform, flux density and rotor bar eddy current density at 350Nm
- Solved with full transient solver inc. rotor rotation (e.g. space harmonics)
- 17% ripple but rotor bar skew isn’t account for in this example
1. eMachine Comparison

Wound Field Synchronous machine

- Similar to Renault Zoe
- 48 slot 8 pole
- 250mm OD
- M250-35A steel
- Rotor winding with 132 turns
1. eMachine Comparison

Wound Field Synchronous machine

- Single layer winding
- Coil pitch - 5 slots
- 6 turns per coil, with 15 strands per turn
- 40% copper slot fill
- 2 parallel paths per phase
1. eMachine Comparison

**Induction machine (IM)**

- Torque and voltage waveform at 350Nm, 500rpm
- 17% torque ripple
- 12% THD on line-line terminal voltage waveform
- Difficult to reduce with WFSM as rotor skewing not feasible
### 1. eMachine Comparison

- All machines designed for equivalent peak performance characteristic
- IM has longer end windings due to winding pattern

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>PM</th>
<th>IM</th>
<th>WFSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active length (mm)</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>End winding overhang (mm)</td>
<td>30</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Total length (mm)</td>
<td>160</td>
<td>200</td>
<td>180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weight</th>
<th>PM</th>
<th>IM</th>
<th>WFSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel (kg)</td>
<td>26.1</td>
<td>33.4</td>
<td>28.17</td>
</tr>
<tr>
<td>Copper (kg)</td>
<td>5.05</td>
<td>13.7</td>
<td>8.5</td>
</tr>
<tr>
<td>Magnet (kg)</td>
<td>2.05</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total (kg)</td>
<td>33.2</td>
<td>47</td>
<td>36.7</td>
</tr>
</tbody>
</table>
1. eMachine Comparison

Peak Performance Comparison

350Nm, 150kW Target

![Graphs showing torque and power vs. speed for different machines (PM, IM, WFSM)]
1. eMachine Comparison

Continuous Performance Comparison

- Stator winding limited on 180°C hotspot
- PM - Magnet limit = 160°C, IM Rotor Bar=220°C, WFSM rotor winding hotspot =180°C
- WFSM limited by rotor temperature, ideally requires rotor cooling
1. eMachine Comparison

PM Machine Efficiency Map

- 96.8% peak efficiency
- Maximum efficiency region from 2-9 krpm
- Large high efficiency region in typical drive cycle area
1. eMachine Comparison

IM Machine Efficiency Map

- 95.5% peak efficiency
- High efficiency region from 7-12krpm
Design 1. eMachine Comparison

WFSM Machine Efficiency Map

- 95.5% peak efficiency
- Maximum efficiency region from 5-10krpm
- High efficiency region at higher torque levels than PM or IM machine
1. eMachine Comparison

Energy Use over a Drive Cycle

- Simple kinematic model used with example EV vehicle parameters

\[ F_{\text{aero}} \]

\[ F_{\text{rolling}} = k_r mg \]

\[ F_{\text{aero}} = \frac{1}{2} \rho v^2 C_d A_f \]

\[ T_{\text{motor}} = k_{\text{dem}} \frac{F_{\text{traction}} \cdot r_\omega}{n_d} \]

\[ \text{Acceleration of Vehicle} = \frac{(F_{\text{traction}} - F_{\text{aero}} - F_{\text{rolling}})}{m} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>1700 kg</td>
</tr>
<tr>
<td>Rolling resistance coeff.</td>
<td>0.0054</td>
</tr>
<tr>
<td>Air density</td>
<td>1.225 kg/m$^3$</td>
</tr>
<tr>
<td>Frontal area</td>
<td>2.81 m$^2$</td>
</tr>
<tr>
<td>Drag Coefficient</td>
<td>0.24</td>
</tr>
<tr>
<td>Wheel radius</td>
<td>0.35m</td>
</tr>
<tr>
<td>Mass correction factor</td>
<td>1.04</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>10:1</td>
</tr>
</tbody>
</table>
1. eMachine Comparison

Energy Use over a Drive Cycle

- WLTP Class 3 Drive Cycle
1. eMachine Comparison

Energy Use over a Drive Cycle

- US06 Drive Cycle
1. eMachine Comparison

Energy Use over a Drive Cycle

- Reduced efficiency = reduced range/increased battery size
- PM gives best efficiency over cycle

<table>
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<th>IPM</th>
<th>IM</th>
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<td>Total Loss - WLTP</td>
<td>255.53Wh</td>
<td>310.25Wh</td>
<td>312.62Wh</td>
</tr>
<tr>
<td>Av. Efficiency - WLTP</td>
<td>94.32%</td>
<td>93.17%</td>
<td>92.79%</td>
</tr>
<tr>
<td>Total Loss - US06</td>
<td>176.09Wh</td>
<td>223.57Wh</td>
<td>214.9Wh</td>
</tr>
<tr>
<td>Av. Efficiency - US06</td>
<td>94.72%</td>
<td>93.39%</td>
<td>93.44%</td>
</tr>
</tbody>
</table>
1. eMachine Comparison

**Dual Motor Solution?**

- Premium EVs are tending to adopt a dual motor topology, one on each axle
- For example, Jaguar i-pace, Audi e-tron, Tesla Model 3/S/X
- Could optimal efficiency and energy use be achieved by using the PM machine on the rear axle and IM machine on the front or vice versa?
- The IM machine could be optimised for higher speed, low torque cruising. While the PM machine could work well for low-medium speed operation and high torque operation
- Tesla have announced a version of the model 3 with this set-up
1. eMachine Comparison

Summary

- **PM machine** offers **improved efficiency** and reduced mass/volume at higher cost.

- Systems aspects, such as improved range or reduced battery mass for the same, with increased efficiency may mean that the **PM motor** gives the lowest overall system cost.

- However, the **PM and IM machines** show **improved efficiencies at different areas of the map** and if a dual motor configuration is used this could be advantageous.

- The **WFSM** has similar performance to the IM however the thermal performance is very constrained on rotor temperature and really some **rotor cooling system is required** in this example.
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Mechanical Analysis
NVH and Mechanical Stress

4. NVH Analysis - Behaviour of Motor + Gearbox
5. Mechanical Stress Analyses (Problem formulation and Solution)
2. Winding Comparison

**Stranded vs Hairpin**

- Hairpin windings are growing in popularity and are used in the Gen2 GM volt and Toyota Prius MY17
- They offer advantages in manufacturing cost and performance repeatability
- However, they also have some disadvantages
- The next section of the comparison looks at hairpin vs. stranded windings

**Toyota Prius MY17**

![Toyota Prius MY17 Winding](image1)

**GM Volt**

![GM Volt Winding](image2)
2. Winding Comparison

**Stranded PM Machine**

- Brushless PM machine
- 48 slot 8 pole IPM
- Double layer magnet – similar to BMW i3
- N42UH magnet
- M250-35A steel
- 250mm OD
- Multi-stranded stator windings
- Step skewed rotor
- 100mm axial length
2. Winding Comparison

Stranded PM Machine

- Single layer winding
- Coil pitch - 5 slots
- 6 turns per coil, with 15 strands per turn
- 40% copper slot fill
- 2 parallel paths per phase
2. Winding Comparison

**Hairpin PM Machine**

- Brushless PM machine
- 48 slot 8 pole IPM
- Double layer magnet – similar to BMW i3
- N42UH magnet
- M250-35A steel
- 250mm OD
- Multi-stranded stator windings
- Step skewed rotor
- 100mm axial length
2. Winding Comparison

Hairpin PM Machine

- Single layer winding
- Coil pitch - 5 slots
- 2 parallel paths per phase
2. Winding Comparison

- CSA of the slot is slightly lower with the hairpin
- Slot fill factor is higher giving lower resistance
- Higher AC loss but lower DC loss with hairpin

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<tr>
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<tbody>
<tr>
<td>Copper Fill Factor</td>
<td>0.4</td>
<td>0.65</td>
</tr>
<tr>
<td>Slot Cross Sectional Area</td>
<td>145.7mm$^2$</td>
<td>130.9mm$^2$</td>
</tr>
<tr>
<td>Conductor Cross Sectional Area</td>
<td>58.29mm$^2$</td>
<td>85.14mm$^2$</td>
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<tr>
<td>Phase resistance</td>
<td>0.0113Ω</td>
<td>0.00773Ω</td>
</tr>
<tr>
<td>DC winding loss @ 100Nm, 8000rpm</td>
<td>1239W</td>
<td>873.6W</td>
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<td>AC winding loss @ 100Nm, 8000rpm</td>
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2. Winding Comparison

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2. Winding Comparison

- Hairpin end windings are more compact on one end due to joins

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<td>25mm - Avg</td>
</tr>
<tr>
<td>Total length</td>
<td>160mm</td>
<td>150mm</td>
</tr>
<tr>
<td>Weight</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>26.1kg</td>
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</tr>
<tr>
<td>Copper</td>
<td>5.05kg</td>
<td>7.36kg</td>
</tr>
<tr>
<td>Magnet</td>
<td>2.05kg</td>
<td>2.05kg</td>
</tr>
<tr>
<td>Total</td>
<td><strong>33.2kg</strong></td>
<td><strong>35.5kg</strong></td>
</tr>
</tbody>
</table>
2. Winding Comparison

Peak Performance Comparison

- Similar peak performance characteristics
- Difference due to slight change in stator slot shape and increased losses at higher speed for the hairpin machine
2. Winding Comparison

Continuous Performance Comparison
- Hairpin gives improved continuous torque at low speed but reduced at higher speeds.
2. Winding Comparison

Stranded Windings Efficiency Map

- 96.8% peak efficiency
- Maximum efficiency region from 2-9krpm
- Large high efficiency region in typical drive cycle area
2. Winding Comparison

Hairpin Windings Efficiency Map

- Higher peak efficiency than stranded machine
- However efficiency at higher speeds is worse
2. Winding Comparison

Energy use over drive cycles

Over both cycles the hairpin machine offers improved efficiency over the stranded design

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<td>95.03%</td>
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2. Winding Comparison

Summary

- Hairpin machine shows generally improved performance across the performance range.
- However, AC losses at higher speeds have the potential to create issues in performance and should be considered from an early stage in the design process.
- Over both cycles, the hairpin machine offers improved efficiency over the stranded design.
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3. Cooling System Comparison

Cooling Systems

Using the IPM hairpin machine we will compare three different cooling methods:

1) Water jacket, e.g. Nissan Leaf, BMW i3
2) Water jacket + Internal Air, e.g. Zytek traction machine, BMW 2225xe series
3) Oil spray cooling, e.g. Toyota Prius
3. Cooling System Comparison

Water Jacket Cooling System

- Spiral water jacket
- 65 degC inlet temperature
- 6.5 l/min coolant flow rate
- EGW 50/50 coolant
- Cooling channels over active machine section only
3. Cooling System Comparison

**Water Jacket + Air Cooling System**

- Air is blown through the duct in the rotor and airgap by a fan
- This air is then recirculated through the housing to use this as a heat exchanger
- The system is sealed and enables rotor cooling

“The innovative traction motor of the BMW 225xe active tourer”, Advanced E-Motor Tech 2017, Dr.-Ing. A. Huber
3. Cooling System Comparison

Oil Spray Cooling

- Oil passed through the shaft and thrown from the shaft onto the inner end winding surface using centrifugal force
- Oil tubes also run over the stator active section and drip oil over the outer surface of the end windings
- A sump collects the oil and passes through a heat exchanger.
- A flow rate of 4 l/min is assumed for the shaft oil and 8 l/min for the stator oil cooling with a 80°C inlet temperature
- This approach is potentially cheaper as it allows the oil cooling system to be shared with the transmission
3. Cooling System Comparison

Modelling Oil Spray Cooling

- Correlations calculate heat transfer of surfaces based on surface area and oil flow rate, velocity and temperature
- Users need to define the flow path of the oil from the nozzle
- We are undertaking various research projects to test oil cooled machines, visualise the oil distribution and develop correlated models
- The types of flow investigated include, axial jets, oil drip, oil mist and oil thrown from rotor using centrifugal forces
Continuous Performance Comparison

- Oil gives best heat transfer
- At higher speeds the air cooling offers the most benefit as the internal flow rate is related to the shaft speed

![Graph showing comparison of torque and power at different speeds for different cooling systems.](image)
3. Cooling System Comparison

**Summary**

- It is tempting to draw generalised conclusions from these sort of studies but it *is often a mistake* to do so.
- Small variations in the specifications and constraints can result in large differences in design decisions.
- System design is very iterative and many different topologies and design decision need to be evaluated during the system optimisation.
- On many occasions the technical trade-offs need to be weighed against other concerns such as risk.
- Using state of the art software, motor design variations can be studied very quickly and easily enabling an optimal motor design and system configuration.
Tutorial Overview

Motor Design
Trade-off Analysis for a BEV traction application
1. eMachine Comparison - PM, IM, Sync
2. Windings Comparison - Hairpin vs Stranded
3. Cooling Comparison - Water Jacket, Internal Air and Oil Spray

Mechanical Analysis
NVH and Mechanical Stress
4. NVH Analysis - Behaviour of Motor + Gearbox
5. Mechanical Stress Analyses - Problem formulation and Solution
**4. NVH Analysis**

**NVH Behaviour of Motor + Gearbox**

- NVH response has typically been left to the latter stages of the design process
- This can be expensive particularly if the motor and gearbox have acceptable NVH characteristics in isolation but problems occur when they are coupled
- Here we compare two interior PM motors for an EV traction application
- Consider the combined NVH response of the motor & gearbox
Motor Specification

Two interior PM (IPM) machines have been designed to meet the given specification

- 12 slot 10 pole design
- 12 slot 8 pole design

Same stator OD & axial length

### Specification

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Torque</td>
<td>160Nm</td>
</tr>
<tr>
<td>Peak Power</td>
<td>70kW</td>
</tr>
<tr>
<td>DC Link Voltage</td>
<td>400Vdc</td>
</tr>
<tr>
<td>Continuous Torque</td>
<td>80Nm</td>
</tr>
<tr>
<td>Stator Outer Diameter</td>
<td>216mm</td>
</tr>
<tr>
<td>Cooling system</td>
<td>TENV</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>12,000rpm</td>
</tr>
</tbody>
</table>
12 slot 10 pole machines have some favourable characteristics however they are well known for NVH issues.

12 slot 8 pole (1.5 slots per pole) machines are a commonly used topology but can exhibit high levels of torque ripple and voltage harmonics.
Design Comparison – Torque Ripple Low Speed

160Nm, 500rpm:

12/10 – 4.78% ripple
12/8 – 4.81% ripple

4. NVH Analysis

Analysis

12 slot 10 pole

12 slot 8 pole
Design Comparison – Torque Ripple High Speed

60Nm, 12,000rpm:

12/10 – 9.1% ripple
12/8 – 51% ripple

Torque ripple at higher speeds is larger for the 12/8 design
4. NVH Analysis

Motor noise mechanisms

1) Torque ripple
   - Equal and opposite torque on rotor and stator

2) Radial forces
   - Act between rotor and stator
   - Forces on rotor cancel out

Forces on stator generate complex force shapes
4. NVH Analysis

Import machine excitations into RomaxDesigner

- Radial Force excitations are of similar magnitude, fundamental is highest for both motors
- 12/10 has slightly higher fundamental (10th Harmonic) in field weakening (at high speed)
4. NVH Analysis

Imported excitation data – 12/10 vs. 12/8

12/10 excitation data

12/8 excitation data
4. NVH Analysis

Comparison of machine response

10th harmonic radial 128 rpm

10th harmonic radial 10500 rpm

24th harmonic radial/TR 1488 rpm

24th harmonic radial/TR 11000 rpm
4. NVH Analysis

Summary

Comparing both traction motors using the excitation data (electromagnetic analysis alone) shows:

- **12/8** machine has highest torque ripple, particularly in field weakening region (high speed)
- **12/10** machines has slightly higher radial force magnitude, particularly in field weakening region

Using the combined Motor-CAD and RomaxDesigner solution we can identify the preferred traction motor for the drivetrain

- **12/8** machine preferred candidate for NVH performance across speed range
- **12/10** machine may be preferred for performance sub-40kph

Analysis of system NVH response is required - Hard to judge based on excitation alone
Tutorial Overview

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5. Mechanical Stress Analysis

Overview

- Objectives
  - Evaluate the stress level
  - Calculate the displacements (deformation)
  - Evaluate the vibration modes
  - Verify the EM integrity against mechanical failure
  - Inspire optimised design for power density (structural mass may count for more than 50% of the overall mass)

- Formulation
  - Load and material
  - Strain – displacement
  - Boundary conditions

- Solution approach
  - Analytic
  - FE

- Post-processing
  - Checks against failure
5. Mechanical Stress Analysis

Flowchart

- **Electromagnetic**
  - Maxwell stress tensor
  - Centrifugal forces (impact on the airgap geometry)

- **Mechanical (structural)**
  - Displacements \( u_i \)
  - Stress tensor \( \sigma_{ij} \)

- **Thermal**
  - Temperature distribution (impact on magnetic properties)

Mechanical Integrity Checks
5. Mechanical Stress Analysis

Formulation

- General [1] 
  \[(u_i\text{=displacement, } \sigma_{ij}\text{= stress tensor, } \Delta T\text{= temperature rise})\]

\[\text{div} \left( \sigma_{ij} \right) + F_i = 0 \]

Deformation model (strain definition)

\[e_{ij} = e_{ij} \left( u_i \text{, } \Delta T \right)\]

Material model

\[\sigma_{ij} = \sigma_{ij} \left( e_{ij} \text{, } \Delta T \right)\]

Boundary conditions (prescribed force or displ.)

\[\sigma_{ij} n_j = p_i \text{ or } u_i = \overline{u}_i\]

Particularisation for linear isotropic elasticity (Hooke’s Law) & small displacements [2]

\[e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)\]

Strain tensor

\[\sigma_{ij} = \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \varepsilon_{kk} \delta_{ij} + \frac{E}{1 + \nu} \varepsilon_{ij} - \frac{E}{1 - 2\nu} \alpha \Delta T \delta_{ij}\]

Material
5. Mechanical Stress Analysis

Formulation: magneto-static vs linear elasticity (2D)

Magnetostatic linear 2D

\[ A_z = A = A(x, y) \]  (vector potential)

Main unknowns:

\[ \Delta A = -\mu j \text{ in } \Omega \]

PDE formulation:

\[ A = 0 \text{ on } \partial \Omega \]

Boundary conditions:

\[ H_x = \frac{B_x}{\mu}, \quad H_y = \frac{B_y}{\mu} \]

Material model:

\[ B_x = \frac{\partial A}{\partial y}, \quad B_y = -\frac{\partial A}{\partial x} \]  (flux density components)

Derived quantities:

\[ \sigma_{xx} = E \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{1 + \nu}{2} \frac{\partial^2 u_y}{\partial x \partial y} + \frac{1 - \nu}{2} \frac{\partial^2 u_x}{\partial y^2} \right), \quad \sigma_{yy} = \sigma_{xy} = 0 \]

Linear elasticity 2D

\[ \mathbf{u} = (u_x, u_y) \]  (displacements)

\[ \begin{cases} \frac{E}{1 - \nu^2} \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{1 + \nu}{2} \frac{\partial^2 u_y}{\partial x \partial y} + \frac{1 - \nu}{2} \frac{\partial^2 u_x}{\partial y^2} \right) = F_{w,x} \quad \text{in } \Omega, \\ \frac{E}{1 - \nu^2} \left( \frac{1 + \nu}{2} \frac{\partial^2 u_x}{\partial x^2} + \frac{1 - \nu}{2} \frac{\partial^2 u_y}{\partial x \partial y} \right) = F_{w,y} \end{cases} \]

\[ \mathbf{u} = \mathbf{0} \quad \text{on } \partial \Omega_1, \quad \sigma \cdot \mathbf{n} = \mathbf{p} \quad \text{on } \partial \Omega_2 \]

\[ \begin{bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1 - \nu \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \end{bmatrix} \]

\[ \varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{xy} = \frac{\partial u_y}{\partial y}, \quad \gamma_{xy} = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \]  (strain components)
5. Mechanical Stress Analysis

Loads

- Centrifugal (volume) forces $\rho \omega^2 r$
- Centrifugal (surface) forces transmitted by PMs to the core......or contact constraints (non-linear!)
- Temperature gradients $\Rightarrow$ thermal stresses
- Initial stresses, e.g. shrink-fit, manufacturing processes (?)
- Electromagnetic normal & tangential (surface) forces (from Maxwell Tensor)

\[ \frac{B_i B_j}{\mu_0} - \frac{|B|^2}{\mu_0} \delta_{ij} \]
In high-speed rotors, centrifugal forces dominate the electromagnetic forces (especially in PM pockets)

\[ p_\mu = \frac{B^2}{2 \mu_0} = \frac{1^2 \times 10^7}{2 \times 4 \pi} \approx 0.4 \text{ MPa} \]

\[ p_\omega = \rho_{PM} \omega^2 r_C h_{PM} \approx \rho_{PM} V^2 \frac{h_{PM}}{r_C} = 8000 \times 100^2 \frac{4}{70} \approx 4.6 \text{ MPa} \]

To avoid contact formulation, PM centrifugal forces can be converted into pressure distributions on two active surfaces 1 & 2 (PM slot “roof”)

\[ V \approx \omega^2 r_C \]

\[ b_{PM} \]

\[ h_{PM} \]

\[ p_{\omega 1} = F_\omega \sin \theta, \quad p_{\omega 2} = F_\omega \cos \theta \]

\[ e_2 = \frac{h_{PM} - h_2}{2}, \quad e_1 = \frac{F_{\omega 2} e_2}{F_{\omega 1}} \]

\[ p_{\omega 2} \approx \frac{F_{\omega 2}}{h_2}, \quad p_{\omega 1} \approx \begin{cases} \text{triangular} & \text{if } e_1 > b_{PM} / 6 \\ \text{trapezoidal} & \text{if } e_1 \leq b_{PM} / 6 \end{cases} \]
5. Mechanical Stress Analysis

PM Contact vs equivalent pressure (Nissan Leaf)

Centrifugal forces on inner PMs only, 6000 rpm

Contact constraints (Solidworks)

Equivalent pressure (Freefem++)

Fairly accurate Von Mises stress $\sigma_{VM}$ around the notch!

$\sigma_{VM}$ not relevant for PMs

$v_{f1} F_{c1}$

$F_{f1}$

$p_{f1}$

$p_{f2}$

IsoValue

0

3.92048

5.66087

8.7913

11.7717

14.6222

17.4246

20.2928

23.1436

26.0052

28.8568

31.7087

34.5605

37.4124

40.2640

43.1156

45.9672

48.8188

51.6703

54.5219

57.3725

60.2241

63.0756

65.9272

68.7788

71.6303

74.4819

77.3335

80.1850

83.0366

85.8882

88.7397

91.5912

94.4428

97.2944

57.55

60.41

63.27

66.13

68.99

71.85

74.71

77.57

80.43

83.29

86.15

89.01

91.87

94.73

97.59

0.0

2.7

5.5

8.2

10.9

13.6

16.4

19.1

23.8

28.0

30.0

32.7

35.5

38.2

40.9

43.7

46.4

50.0

54.6

58.2

61.8

65.3

68.9

72.5

76.1

79.7

83.4

87.0

90.6

94.2

97.8

v_{f1} F_{c1}$

$F_{f1}$

$p_{f1}$

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23.8

28.0

30.0

32.7

35.5

38.2

40.9

43.7

46.4

50.0

54.6

58.2

61.8

65.3

68.9

72.5

76.1

79.7

83.4

87.0

90.6

94.2

97.8
If the iron bridge is very thin, pressure distribution $p_{\omega 2}$ is not uniform!

Need to include the contribution of the PM to the bridge stiffness to find $p_{\omega 2}$
### 5. Mechanical Stress Analysis

**Materials**

- Linear elastic model adequate for ductile materials below the yield stress
- PM materials (brittle) may have different Young’s modulus under tensile and compression stress
- Laminated cores are anisotropic....but do you know the properties along z-axis?!?
- Composite materials (fibre) require anisotropic models
- Progressive collapse analysis (if required) needs non-linear models for ductile materials (plasticity)

![Stress-Strain Curve](image.png)

- Initial unloading path is linear! (unlike B-H curve)
- Mild steel (ductile, symmetrical)
- Cast iron (brittle, unsymmetrical)
### 5. Mechanical Stress Analysis

**Strain model and load setup**

- Non-linearity (geometric) and need for iterative solution can arise from the deformation model, even with linear materials
- “Small displacements” ⇒ the relationship $e_{ij}(u_i)$ is linearised
- Loads can be applied to either the undeformed or deformed configuration (⇒ geometric non-linearity)

<table>
<thead>
<tr>
<th>Loads evaluated with:</th>
<th>Small displacements</th>
<th>Large displacements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Un-deformed configuration</strong></td>
<td>Checks against failure in “rated” conditions <strong>(no iteration)</strong></td>
<td>-----</td>
</tr>
<tr>
<td><strong>Deformed configuration</strong></td>
<td>Instability Contact Progressive collapse <strong>(iterative)</strong></td>
<td>Instability Contact Progressive collapse <strong>(iterative)</strong></td>
</tr>
</tbody>
</table>

---

**Mechanical Stress Analysis**

\[
\begin{align*}
F_x & \quad F_y \\
\text{Load} & \quad \text{Load} \\
L & \quad L \\
\end{align*}
\]

- $u_x < L$ \(\Rightarrow\) $u_y \approx 0$
- $u_y << u_x$
- No limitations on $u_x$ and $u_y$
**5. Mechanical Stress Analysis**

**Reduction to 2D models: plane stress vs plane strain**

- **Plane stress:** thin disc & loads only in the \( x-y \) plane \( \Rightarrow \sigma_z \approx 0 \)
  
  \[ F(x, y) \]
  
  \[ \sigma_z = \frac{E}{(1 + \nu)(1 - 2\nu)} (v\varepsilon_x + v\varepsilon_y + (1 - v)\varepsilon_z) = 0 \]
  
  \[ \varepsilon_z = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y) \neq 0 \]
  
  \[ u_z = \int_A \varepsilon_z \, dz \neq 0! \]
  
- **Plane strain:** infinitely long prisms & constant load along \( z \) \( \Rightarrow \varepsilon_z=0 \)
  
  \[ F(x, y) \]
  
  \[ \varepsilon_z = \frac{1}{E}(\sigma_x + \sigma_y) = 0 \]
  
  \[ \sigma_z = v(\sigma_x + \sigma_y) \neq 0 \]
  
  \[ N = \int_A \sigma_z \, dA \neq 0! \]
  
- **Generalised plane strain:** as the previous one but with \( \varepsilon_z = \varepsilon_{z0} = \text{const.} \) so \( N=0 \)
  
  \[ F(x, y) \]
  
  \[ \varepsilon_{z0} = -\frac{v}{1-v} \int_A (\sigma_x + \sigma_y) \, dz \]
  
  \[ \sigma_z = v(\sigma_x + \sigma_y) + E\varepsilon_{z0} \neq 0 \]
  
  \[ N = \int_A \sigma_z \, dA = 0! \]

*The solution is accurate only in the mid section \( z=0 \) far away from the ends*
Boundary conditions

- In static models, appropriate boundary conditions (BCs) are to be set in order to stop any rigid motion (for non-singular stiffness matrix).
- BCs must represent the real constraints without introducing extra stiffness.
- 2D models cannot include shaft ends and bearings & coupling, so they need alternative BCs to stop rigid movement of the rotor.
  - Periodic BCs + zero average tangential displacement at the inner radius (viable only if the machine periodicity is a submultiple of $2\pi$).
  - Zero average horizontal, vertical and tangential displacements at the inner radius (may be tricky to enforce!).
  - The zero average tangential displacement condition may be replaced by zero tangential displacement.
  - The shrink-fit shaft/hub is represented by a constant pressure at the inner surface of the hub.
5. Mechanical Stress Analysis

Boundary conditions: 2D models

- Periodic BCs (pole pitch or machine periodicity)
  - Rotation still allowed!
  - Prevents rotation!
  - \( p = \text{pressure (shaft-hub shrink fit)} \)

- General BCs (... \( 2\pi \) -periodicity, e.g. machines with UMP...)

\[
\begin{align*}
\int_0^{2\pi} u_x (\theta) \, d\theta &= 0 \\
\int_0^{2\pi} u_y (\theta) \, d\theta &= 0 \\
\int_0^{2\pi} u_\theta (\theta) \, d\theta &= 0 \\
\int_0^{2\pi} \left( u_y (\theta) \cos \theta - u_x (\theta) \sin \theta \right) \, d\theta &= 0
\end{align*}
\]

Zero average displacements on \( \Gamma \)
5. Mechanical Stress Analysis

Verification of critical conditions

- Static failure
  - static loads or limited number of cycles ($\leq 1000$)

- Fatigue
  - Varying (periodic) loads with high number of cycles ($>10^3$)

- Decompression / sliding in shrink fits

- Critical speeds (rotor-dynamics)

- Maximum displacement
  - Airgap clearance

- (Instability)
  - *Slender geometries (e.g. stators with thin back-iron depth)*

- (Plastic collapse)
  - *Assess ultimate strength for increasing loads*
5. Mechanical Stress Analysis

Static failure

- A generic 3D stress combination (tensor) is fully represented in terms of principal stresses \( \{\sigma_I, \sigma_{II}, \sigma_{III}\} \).
- Needs a criterion to compare a generic stress combination with data from standard uni-axial (1D) tensile stress tests.
- Formulations depend on the behaviour of each material.

**Ductile** (metal alloys - except when \( \sigma_I \approx \sigma_{II} \approx \sigma_{III} \geq 0! \))

\[
\sigma_{VM} = \sqrt{\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2}{2}} \leq \frac{\sigma_y}{\eta}
\]

\( \eta = \) safety factor (depends on model and material uncertainties)

For ductile behaviour, \( \sigma_{VM} = \sigma_y \) is just a conventional limit.

**Brittle** (sintered, ceramics, metals with \( \sigma_I \approx \sigma_{II} \approx \sigma_{III} > 0! \))

\[
\max \{\sigma_I, \sigma_{II}, \sigma_{III}\} \leq \frac{\sigma_I}{\eta} \quad \text{and} \quad \min \{\sigma_I, \sigma_{II}, \sigma_{III}\} \geq -\frac{\sigma_c}{\eta}
\]
5. Mechanical Stress Analysis

Fatigue

- Fatigue life is affected by many aspects [3]-[5] (material, surface finishing, notches, min/max stress ratio $R$)

- Test data refer to $R=-1$: Woehler Curve, 50% survival probability

- Classic approach based on nominal stress $\sigma_n$ & concentration factor

- Criteria for planar or 3D stress combinations (e.g. Sines) or based on Fracture Mechanics (Stress Intensity Factor) [5]

\[
K_f \sigma_n \leq \frac{\sigma_{\infty,R}}{\eta K_1 K_2 K_3}
\]

For centrifugal forces in on-off cycles, $R=0!$

\[
\sigma_{\infty,0} = \frac{2 \sigma_{\infty,-1} \sigma_R}{\sigma_R - \sigma_{\infty,-1}}
\]
5. Mechanical Stress Analysis

Analytic Models

Features and motivation
- Easy to integrate with magnetic lumped-parameter models
- Useful for initial design and optimisation
- Cross-check FE results (!!)

Available options
- Exact solutions for the Equations of the Theory of Elasticity are available only for simple geometries - mainly 2D and axisymmetric (IM solid rotors, SPM rotors)
- Beam and plate theories provide additional tools [2] to set up approximate models (e.g. for iron bridges in IPM rotors)
- Complex load configurations are treated with superposition
- Stress concentration charts (e.g. Peterson’s) are available for notches (Fatigue)
5. Mechanical Stress Analysis

Rotating disc vs cylinder: overview

- The classic solution with $\varepsilon_z=0$ and $L=\infty$ needs a correction constant $\sigma_{z0}$ to produce zero axial resultant force $N$ at the ends.
- The adjusted solution for the cylinder predicts identical displacements to plane-stress solution ($\sigma_z=0$) for discs.
- Stress distributions are slightly different: the cylinder has lower $\sigma_{VM}$ but higher individual principal stress values (important for brittle PMs).

![Diagram showing stress distributions for solid and hollow disks and cylinders.](image)
5. Mechanical Stress Analysis

Rotating disc vs cylinder: Stresses and displacements

Cylinder (L<∞ & N=0)

\[ \sigma_r = \frac{1}{8} \left( 3 - 2v \right) \rho \omega^2 r_e^2 \left( 1 - \left( \frac{r}{r_e} \right)^2 \right) \]

\[ \sigma_\theta = \frac{1}{8} \left( 1 - 2v \right) \rho \omega^2 r_e^2 \left( 1 - \frac{1}{3 - 2v} \left( \frac{r}{r_e} \right)^2 \right) \]

\[ \sigma_z = \frac{1}{4} \frac{\rho \omega^2}{1 - 2v} r_e^2 \left( 1 - 2 \left( \frac{r}{r_e} \right)^2 \right) \]

\[ u_{re} = \frac{1}{4E} \left( 1 + v \right) \rho \omega^2 r_e^3 \]

\[ a = \frac{r_e}{r_i} \]

Disc & Cylinder

\[ \sigma_r = \frac{1}{8} \left( 3 - 2v \right) \rho \omega^2 r_e^2 \left( 1 + \frac{1}{a^2} - \left( \frac{r_i}{r} \right)^2 - \left( \frac{r}{r_e} \right)^2 \right) \]

\[ \sigma_\theta = \frac{1}{8} \left( 1 - 2v \right) \rho \omega^2 r_e^2 \left( 1 + \frac{1}{a^2} + \left( \frac{r_i}{r} \right)^2 - \frac{1}{3 - 2v} \left( \frac{r}{r_e} \right)^2 \right) \]

\[ \sigma_z = \frac{1}{4} \frac{\rho \omega^2}{1 - 2v} r_e^2 \left( 1 + \frac{1}{a^2} - 2 \left( \frac{r_i}{r} \right)^2 \right) \]

\[ u_{ri} = \frac{(1 + v) (3 - 2v)}{4E} \left( \frac{a^2 - \frac{1 - 2v}{3 - 2v}}{r^3} \right) \rho \omega^2 r_i^3 \]

\[ u_{re} = \frac{(1 + v) (3 - 2v)}{4E} \left( \frac{1 - 2v}{3 - 2v} + \frac{1}{a^2} \right) \rho \omega^2 r_e^3 \]
5. Mechanical Stress Analysis

Rotating cylinder: end effects

- The solution with $\varepsilon_z=$const and $N=0$ is only valid in the mid section and for $L/R$ aspect ratio not too small.
- End effects result in a $\tau_{rz}$ distribution that dies away far from the ends.
- In “short” cylinders $L/R<2$, the solution for $\sigma_r$ and $\sigma_\theta$ approaches the one for discs, but $\sigma_z \neq 0$ is still present!

Example with $L/R=2$
(steel, $R=50$ mm, $n=12,000$ rpm)

Finite $L$ (FE)
$L=\infty$, $N=0$ (Analytic)
Disc / cylinder with pressure load

- Pressure is generated by shrink-fit or pre-stressed bandage
- Solution for discs predicts $\varepsilon_z=$const. so it applies to cylinders of finite length too and is exact (no end effects)!

\[
\sigma_r = p_e \frac{a^2}{a^2 - 1} \left( \frac{r_i}{r} \right)^2 - 1
\]

\[
\sigma_\theta = -p_e \frac{a^2}{a^2 - 1} \left[ 1 + \left( \frac{r_i}{r} \right)^2 \right]
\]

The multi-layer model requires **stiffness coefficients** $k_{ri}$, $k_{ri,i}$, $k_{re,i}$, $k_{ri,e}$ and $k_{re,e}$
Disc / cylinder with thermal effects

- Thermal expansion is important in multi-layer models to assess the performance of a shrink-fitted sleeves
- Thermal gradients lead to additional (thermal) stresses that need to be considered
- The rotor temperature distribution may be axisymmetric but depends on $z$ (heat transfer towards the shaft ends): 2D models set in the mid plane do not capture this aspect
- Uniform temperature rise $\Delta T$ leads to expansion only (no additional stress): this ideal scenario could be considered in the first design of the sleeve/bandage system

$$u_T, re = \alpha \Delta T \ r_e$$

### Mechanical Stress Analysis

5. **Mechanical Stress Analysis**

- $\sigma_r = \sigma_\theta = \sigma_z = 0$
- $u_T, re = \alpha \Delta T \ r_e$

---

**Solid**

- $r_e = 0$
- $\Delta T$
- $u_{re}$

**Hollow**

- $r_i = 0$
- $\Delta T$
- $u_{ri}$

$$\sigma_r = \sigma_\theta = \sigma_z = 0$$

$$u_T, r_i = \alpha \Delta T \ r_i$$

$$u_T, re = \alpha \Delta T \ r_e$$
5. Mechanical Stress Analysis
Multi-layer Rotating Cylinder Models

- Gives useful insight into SPM rotors with bandage / sleeve

**but**

- Segmented PMs can be replaced by an homogeneous layer only if the hoop stress $\sigma_z$ is negative (compression, i.e. sufficient pre-stress in the bandage)
- Plane strain ($\varepsilon_z=\text{const.}$) $\Rightarrow$ results valid on the mid section $z=0$
- Do not capture tangential stress $\tau_{rz}$ in the PM ends (potentially responsible for PM cracks)
- Do not capture stress concentration in the sleeve near pole gaps, if present
The model may adopt a constant piece-wise temperature profile \( \{ \Delta T_c, \Delta T_{PM}, \Delta T_s \} \) according to eddy current losses in each layer.

Displacement compatibility: \( r_1^{(c)} + u_{r1}^{(c)} = r_1^{(PM)} + u_{r1}^{(PM)} \) and \( r_2^{(PM)} + u_{r2}^{(PM)} = r_2^{(s)} + u_{r2}^{(s)} \)

\[
\begin{align*}
    r_1^{(c)} - k_{r,e}^{(c)} p_1 + u_{\omega, re}^{(c)} + u_{T, re}^{(c)} &= r_1^{(PM)} + k_{r,i}^{(PM)} p_1 - k_{r,i}^{(PM)} p_2 + u_{\omega, ri}^{(PM)} + u_{T, ri}^{(PM)}, \\
    r_2^{(PM)} + k_{r,e}^{(PM)} p_1 - k_{r,e}^{(PM)} p_2 + u_{\omega, re}^{(PM)} + u_{T, re}^{(PM)} &= r_2^{(s)} + k_{r,i}^{(s)} p_2 + u_{\omega, ri}^{(s)} + u_{T, ri}^{(s)}
\end{align*}
\]

- Once \( p_1 \) and \( p_2 \) are found, stress distributions in each layer are found by super-position of stress contributions from \( \omega, p_1 \) and \( p_2 \).
- Check different conditions (\( \omega, \Delta T_k \)) for the (worst-case scenario).
- **If \( p_1 < 0 \) or \( p_2 < 0 \) or \( \sigma_\theta > 0 \) (for segmented magnets) the solution is not valid!**
5. Mechanical Stress Analysis

Multi-layer Rotating Cylinder Models: Some Examples

Stress sensitivity analysis in 500W, 400krpm SPM machine with $D_{PM}=9.4\text{mm}$, $D_{sleeve}=11.6\text{mm}$: impact of speed, fit interference, PM radius (from [6])

![Graphs showing stress analysis results for PMs and Sleeve.]
Multi-layer Rotating Cylinder Models: Some Examples

Stress sensitivity analysis in a 1.12MW, 18krpm SPM machine with $D_{PM} \approx 176$ mm, $D_{sleeve} \approx 190$ mm: impact of speed and temperature (from [7])
5. Mechanical Stress Analysis

Lumped-stiffness models (IPM rotors)

- Usually, iron bridges in different layers work in series (PMs are only in contact with the magnet slot “roof”)
- The analytic model in [8] assumes rigid-body islands (pole shoes) connected with 1D stiffness elements (bridges)
- May be enhanced by considering additional bending stiffness of lateral bridges (...likely to require 2nd-order beam theory due to the interaction bending - axial resultant)
5. Mechanical Stress Analysis

Lumped-stiffness models (IPM rotors)

- Rigid radial displacement of the pole imposes the compatibility of radial and tangential displacements in the thin bridges
  \[ u_t = u_r \cos \theta \]
- Stiffness coefficients
  \[ k_r = \frac{E t_r}{h_r}, \quad k_t = \frac{E t_t}{h_t} \]
- Radial equilibrium
  \[ \begin{align*}
  F_c &= F_r + 2 F_t \cos \theta \\
  F_t &= k_t u_t 
  \end{align*} \]
- Solution for forces and stresses [8]

\[
\begin{align*}
  F_r &= \frac{F_c}{1 + 2 \frac{k_t \cos^2 \theta}{k_r}} \\
  F_t &= \frac{F_c \cos \theta}{\frac{k_r}{k_t} + 2 \cos^2 \theta}
\end{align*}
\]

- Stress concentration factors \( K_{tr} \) & \( K_{tt} \) depend on the notch geometry

\[
\begin{align*}
  \sigma_{rn} &= \frac{F_r}{t_r} \\
  \sigma_{tn} &= \frac{F_t}{t_r}
\end{align*}
\]

\[
\begin{align*}
  \sigma_r &= K_{tr} \sigma_{rn} \\
  \sigma_t &= K_{tt} \sigma_{tn}
\end{align*}
\]
5. Mechanical Stress Analysis

Lumped-stiffness models (IPM rotors)

- V-shape pole shoes can be treated as part of an elastic outer ring suspended with elastic spokes to an inner ring [9]
- The shear and bending stiffness of the lateral bridges is ignored
- The set of resultant normal forces transmitted by spokes are then converted into equivalent uniform pressure on the rings
- Spokes and rings are then assumed to carry normal forces only
Lumped-stiffness models (IPM rotors)

- The resulting analytic equations are convoluted (see [9]).
- The model exhibits good accuracy for moderate PM angle and for the stress in the lateral bridges.
- The stress in the central bridge is underestimated (the model ignores the variable moment of inertia of the pole shoe!)

Analysis results for 2007 Toyota Camry motor (with central bridge) [5]
5. Mechanical Stress Analysis

Lumped-stiffness models (IPM rotors)

- The model may be improved to include parasitic bending moments in the inner/outer rings caused by the spokes.

- Stress concentration around notches and fillets can be evaluated using stress concentration factors from tables [4].
5. Mechanical Stress Analysis

Conclusions

- Linear-elasticity, small-displacements formulation is the standard setup for stress analysis of EM rotors
- Interaction between PMs and rotor stack can be represented by
  - Setting non-penetration contact constraints (non-linear analysis)
  - Using equivalent pressure distributions (linear analysis): correction needed for thin iron bridges
- 2D model features
  - Results are valid only far away from the rotor ends
  - In case of $2\pi$-periodicity (e.g. UMP), boundary conditions are not easy to set
- Verification of mechanical integrity needs appropriate stress metrics depending on
  - Material behaviour (ductile / brittle)
  - Type of loading (static / fatigue)
- Analytic models are available for SPM and IPM, with some limitations
  - Difficult to take into account pole gaps in SPM
  - In IPM rotor bridges, they only predict the average stress level
- *Cross-check FE results with an (even crude) analytic model!*
5. Mechanical Stress Analysis

References


Thank you for your Attention!

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